Question	Scheme	Marks	AOs
1 (a)	Attempts $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times 16\sin t\cos t$ and uses $\sin 2t = 2\sin t\cos t$	M1	2.1
	Correct expanded integrand. **Usually** for one of $(R)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\ dt$ $(R)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\ dt$ $(R)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\ dt$	A1	1.1b
	Attempts to use $\cos 4t = 1-2\sin^2 2t = \left(1-8\sin^2 t\cos^2 t\right)$	M1	1.1b
	$R=\int_0^a 8-8\cos 4t+48\sin^2 t\cos t\ dt$ *	A1*	2.1
	Deduces $a=\frac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8-8\cos 4t+48\sin^2 t\cos t\ dt = 8t-2\sin 4t+16\sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t-2\sin 4t+16\sin^3 t\right]_0^{\frac{\pi}{4}} = 2\pi+4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
		(9 marks)	
Notes:			

(a) Condone work in another variable, say $\theta \leftrightarrow t$ if used consistently for the first 3 marks

M1: For the key step in attempting $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times 16\sin t\cos t$ with an attempt to use $\sin 2t = 2\sin t\cos t$ Condone slips in finding $\frac{dx}{dt}$ but it must be of the form $k\sin t\cos t$

E.g. I $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times k\sin t\cos t=(4\sin t\cos t+3\sin t)\times k\sin t\cos t$

E.g. II $y.\frac{dx}{dt}=(2\sin 2t+3\sin t)\times k\sin t\cos t=(2\sin 2t+3\sin t)\times \frac{k}{2}\sin 2t$

A1: A correct (expanded) integrand in t. Don't be concerned by the absence of $\int$ or dt or limits

$(R)=\int \underline{\underline{48\sin^2 t\cos t+16\sin^2 2t}}\ dt$ or $(R)=\int \underline{\underline{48\sin^2 t\cos t+64\sin^2 t\cos^2 t}}\ dt$

but watch for other correct versions such as $(R)=\int \underline{\underline{24\sin 2t\sin t+16\sin^2 2t}}\ dt$

M1: Attempts to use $\cos 4t = \pm 1 \pm 2\sin^2 2t$ to get the integrand in the correct form.

If they have the form $P\sin^2 2t$ it is acceptable to write $P\sin^2 2t = \frac{P}{2}(\pm 1 \pm \cos 4t)$

If they have the form $Q\sin^2 t\cos^2 t$ sight and use of $\sin 2t$ and/or $\cos 2t$ will usually be seen first.

There are many ways to do this, below is such an example

$$Q\sin^2 t\cos^2 t = Q\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right) = Q\left(\frac{1-\cos^2 2t}{4}\right) = Q\left(\frac{1}{4} - \frac{\cos^2 2t}{4}\right) = Q\left(\frac{1}{4} - \frac{1+\cos 4t}{8}\right)$$

Allow candidates to start with the given answer and work backwards using the same rules. So expect to see $\cos 4t = \pm 1 \pm 2\times\sin^2 2t$ or $\cos 4t = \pm 2\times\cos^2 2t \pm 1$ before double angle identities for $\sin 2t$ or $\cos 2t$ are used.

A1*: Proceeds to the given answer with correct working. The order of the terms is not important.
Ignore limits for this mark. The integration sign and the dt must be seen on their final answer.
If they have worked backwards there must be a concluding statement to the effect that they know that they have shown it. The integration sign and the dt must also be seen

E.g. Reaches $\int 48\sin^2 t\cos t + 64\sin^2 t\cos^2 t \, dt$

Answer is $\int 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt$

$$= \int 8 - 8\left(1 - 2\sin^2 2t\right) + 48\sin^2 t\cos t \, dt$$

$$= \int 16\sin^2 2t + 48\sin^2 t\cos t \, dt$$

$$= \int 64\sin^2 t\cos^2 t + 48\sin^2 t\cos t \, dt$$

which is the same, ✓

B1: Deduces $a = \frac{\pi}{4}$. It may be awarded from the upper limit and can be awarded from (b)

(b)

M1: For the key process in using a correct approach to integrating the trigonometric terms.
May be done separately.
There may be lots of intermediate steps (e.g. let $u = \sin t$).
There are other more complicated methods so look carefully at what they are doing.

$\int 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = ... \pm P\sin 4t \pm Q\sin^3 t$ where P and Q are constants

A1: $\int 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = 8t - 2\sin 4t + 16\sin^3 t(+c)$

If they have written $16\sin^3 t$ as $16\sin t^3$ only award if further work implies a correct answer.
Similarly, 8t may be written as 8x. Award if further work implies 8t, e.g. substituting in their limits.
Do not penalise this sort of slip at all, these are intermediate answers.

M1: Uses the limits their a and 0 where $a = \frac{\pi}{6}, \frac{\pi}{4}$ or $\frac{\pi}{3}$ in an expression of the form $kt \pm P\sin 4t \pm Q\sin^3 t$ leading to an exact answer. Ignore evidence at lower limit as terms are 0

A1: CSO $2\pi + 4\sqrt{2}$ or exact **simplified** equivalent such as $2\pi + \frac{8}{\sqrt{2}}$ or $2\pi + \sqrt{32}$.

Be aware that $\int_0^{\frac{\pi}{4}} 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = 8t \underline{\underline{+\lambda}} \sin 4t + 16\sin^3 t(+c)$ would lead to the correct answer but would only score M1 A0 M1 A0

Question	Scheme	Marks	AOs
2	$y=\frac{(x-2)(x-4)}{4\sqrt{x}}=\frac{x^2-6x+8}{4\sqrt{x}}=\frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\,dx=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\frac{32}{10}-8+8\right)-\left(\frac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\frac{16}{5}-\frac{12}{5}\sqrt{2}$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\left(\text{or } \frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
			(6 marks)
Notes:			

M1: Correct attempt to write $\frac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\frac{1}{4}x^{2-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\frac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\ x^{\frac{1}{2}}\to x^{\frac{3}{2}},\ x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\frac{1}{4}\times\frac{2}{5}x^{\frac{3}{2}+1}-\frac{1}{2}\times\frac{2}{3}x^{\frac{1}{2}+1}-\frac{2}{3}x^{\frac{1}{2}+1}+2\times2x^{\frac{1}{2}}$

or as e.g. $\frac{1}{4}\left(\frac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{10}\times4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times4^{\frac{1}{2}}-\frac{1}{10}\times2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

See overleaf for integration by parts and integration by substitution.

Integration by parts:

$\int \frac{(x-2)(x-4)}{4\sqrt{x}}\,dx = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx} = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\,dx}$ $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} \underline{- \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}}$ Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	dM1 A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$ Area $R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$ $\left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

oe e.g. $\alpha\left(x^2 - 6x + 8\right)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int (px+q)x^{\frac{1}{2}}\,dx$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right)dx = \ldots$ or e.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}}\,dx$

If they expand then at least one term requires $x^n \to x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a "changed" function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Attempts at integration by parts "the other way round" should be sent to review.

Integration by substitution example:

$u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$	M1 A1	1.1b 1.1b
$=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$	M1	2.2a
$\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\ \left(\text{or } \frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

dM1: Expands the bracket and integrates $u^n\to u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4\to u^5,\ u^2\to u^3,\ k\to ku$

A1: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.